Math 10–C

Relations & Functions

Lesson 1 – Introduction to Relations & Functions

TERMINOLOGY

Relation – a rule that associates the elements of one set with the elements of another set (a set of ordered pairs).
ie.) {(2,4), (4,8), (6,12)} is a relation
{(boy, 11)} is a relation
{(8:37, bell)} is a relation
{(beef, $17)} is a relation

Set – a collection of distinct objects

Element – One distinct object in a set.

ie.) {2, 4, 6, 8} = a *set* of natural numbers with *elements* 2, 4, 6, and 8.

When we represent relations using numbers, a relation is a set of ordered pairs. The elements in the relation are the numbers that represent specific coordinate points on a Cartesian plane.

Relations can be represented in a number of ways:

1. Table of Values
2. Graphs
3. Arrow Diagrams (mapping diagram)
4. Equations
5. In Words (description)

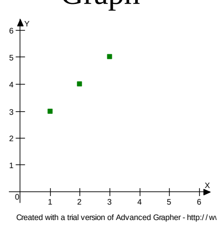
Example #1 – Represent the following relation in 4 different ways:

{(1,3), (2,4), (3,5)} **Recall: (*x*, *y*)**

Table of Values

x	***y***
1	3
2	4
3	5

Graph



Equation

$y = x + 2$

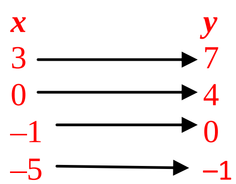
Words

y is always equal to the value of *x* plus two.

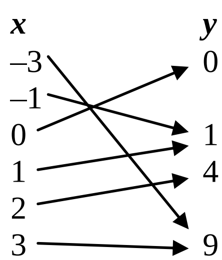
Another way to represent a relation is by drawing an "arrow diagram" or "mapping diagram". Each element represented by an *x* (domain discussed in L2) in the coordinate points is matched with an element represented by a *y* (range discussed in L2) coordinate.

eg.) Draw an arrow diagram for the following relation:

{(3,7), (0,4), (–1,0), (–5,–1)}



eg.) Represent the following arrow diagram in the form of an *equation*:



$y = x^2$

Ex. Animals can be associated with the classes they are in.

Animal	**Class**
Ant	Insect
Eagle	Aves
Snake	Reptilia
Turtle	Reptilia
Whale	mammalia

a. Describe this relation in words.

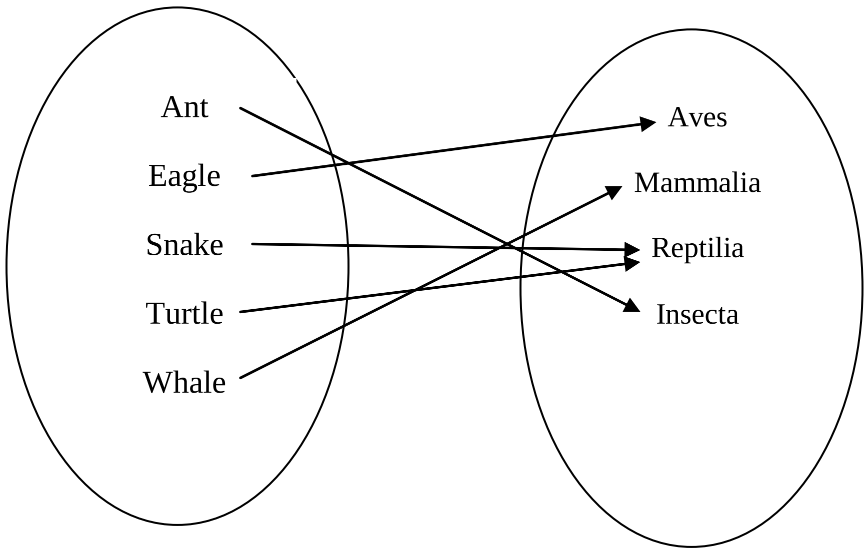
The relation shows the association "belongs to the class" between a set of animals and a set of classes.

b. Represent this relation:

i) as a set of ordered pairs

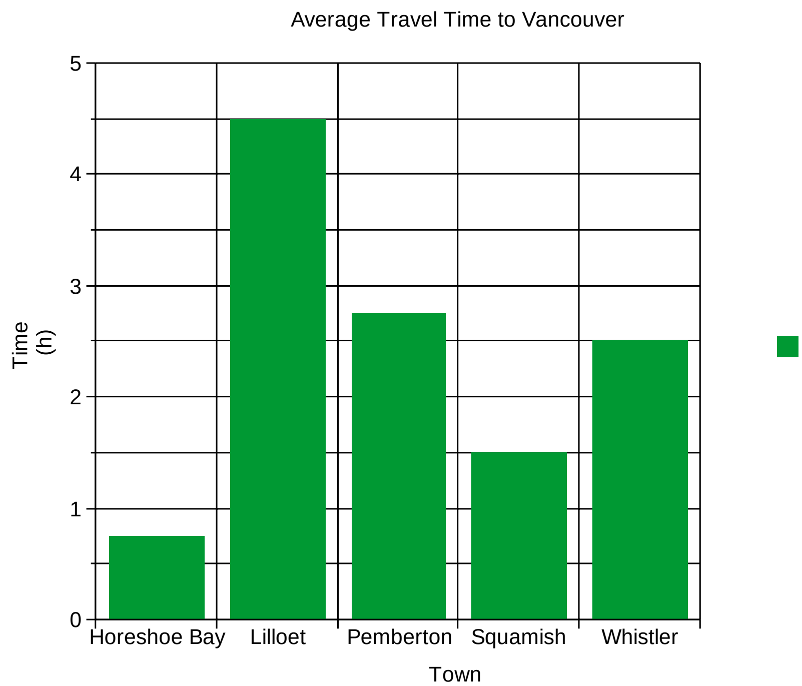
{(ant, insect), (eagle, aves), (snake, reptilia), (turtle, reptilia), (whale, mammalia)}

ii) as an arrow diagram:



When the elements of either one or both sets in a relation are in fact numbers, the relation can be represented as a bar graph as well (not always line graphs).

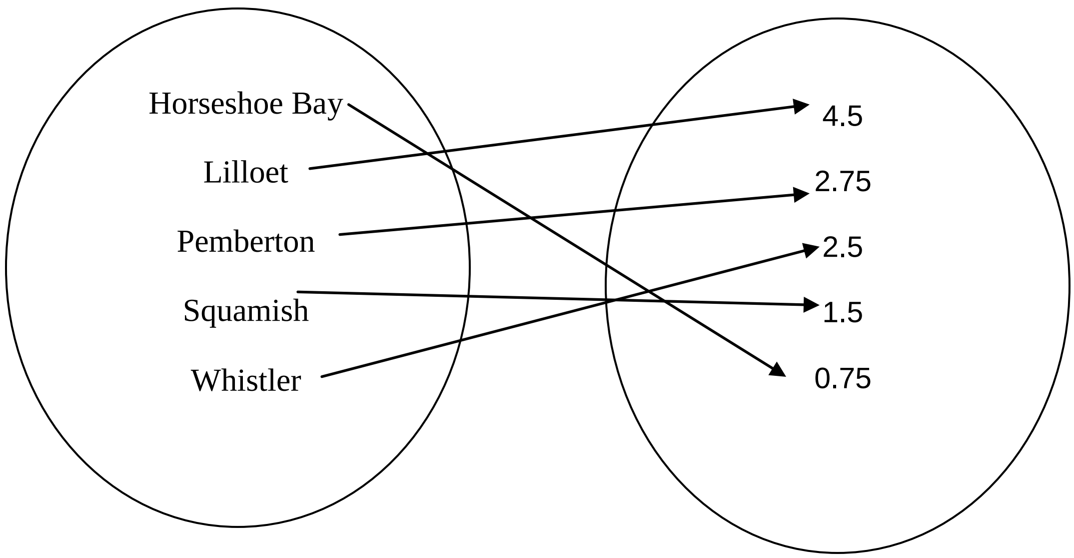
Ex. Different towns in British Columbia can be associated with the average time, in hours, it takes to drive to Vancouver. Consider the relation represented by the following graph.



a) As a table

Town	Average Time(h)
Horseshoe Bay	0.75
Lilloet	4.5
Pemberton	2.75
Squamish	1.5
Whistler	2.5

b) as an arrow diagram:



Relation vs. Function

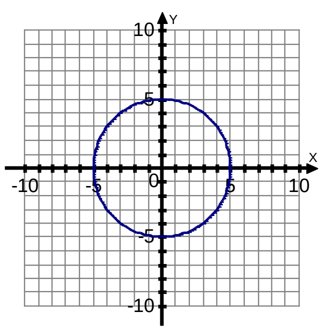
Recall: A relation is a set of ordered pairs. A relation produces one or more output numbers for every valid input number.

eg.) {(–2, 3), (3, 4), (4, 7), (–2, 6)}

A function is also a set of ordered pairs, however, **for every valid input number, there is only ONE output number.**

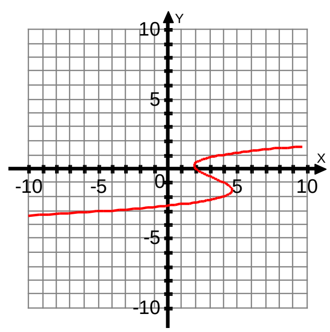
eg.) {(0, 2), (2, 6), (3, 8), (4, 10)

In order to determine if a graph is a relation, or whether it is a function, we can use what we call the "Vertical Line Test". This means that if we draw a vertical line anywhere on the graph, and it intersects the graph ONLY once, then the relation is a function. Any more than 1 makes the graph a relation.



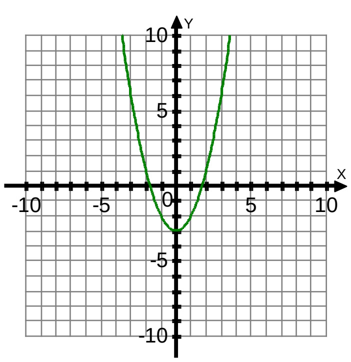
Created with a trial version of Advanced Grapher - http://

Relation



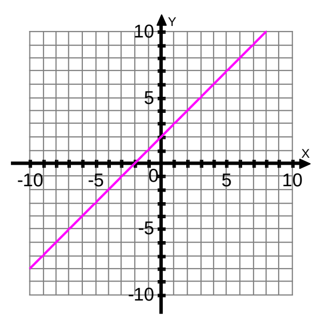
Created with a trial version of Advanced Grapher - http://

Relation



Created with a trial version of Advanced Grapher - http://

Function



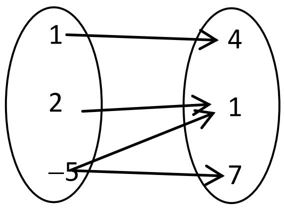
Created with a trial version of Advanced Grapher - http://

Function

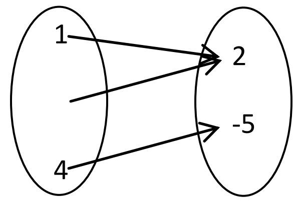
Are the following functions?

1. {(2, –1), (5, 1), (–5, 1)}   Yes

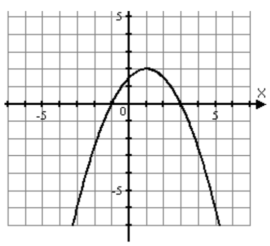
2. {(6, 3), (6, –5)}   No

3. 

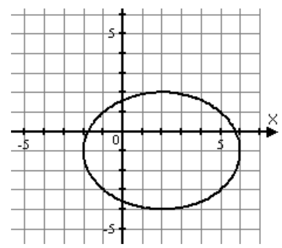
No

4. 

Yes

5. 

Yes

6. 

No

Assignment: Pg. 262–263 #3–8, 10

Pg. 294 #6, 8 (no Domain & Range)

Lesson 2 – Interpreting Graphs

Many times when interpreting graphs, we imagine that our ***x*** and ***y*** axes represent some specific everyday variables. These variables are known as **dependent** and **independent** variables.

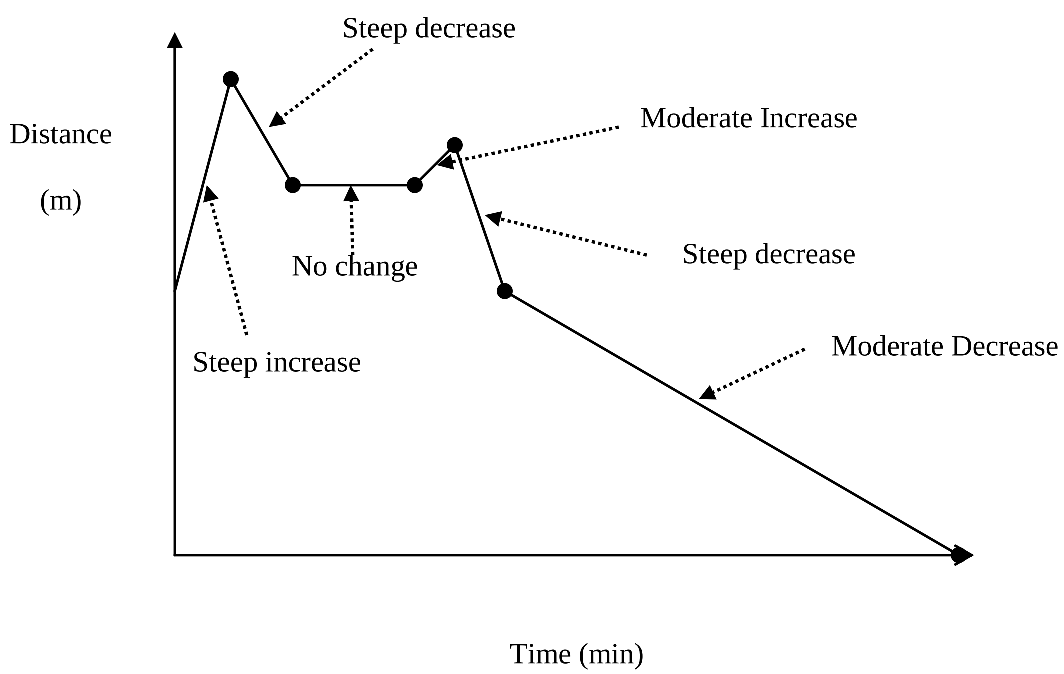
Dependent Variable → the variable whose value is dependent upon or determined by another variable.
→the variable that is ALWAYS on the **y–axis**!

*** Common dependent variables include distance, speed, height, and temperature.*

Independent Variable→ in a relation, the variable whose value may be freely chosen and upon which the value(s) of the other variable depend.
→the variable that is ALWAYS on the **x–axis!**

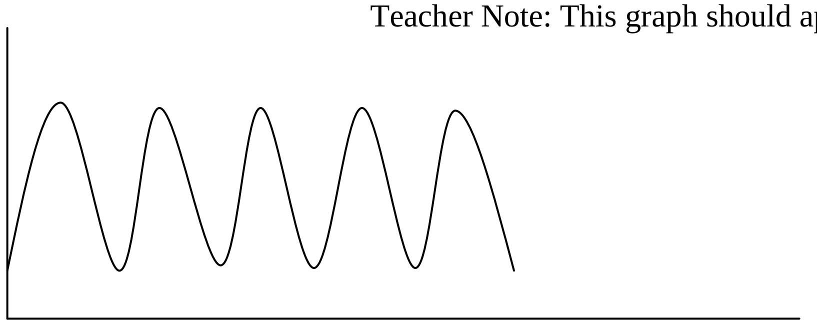
*** The most common independent variable used in mathematics and science is TIME.*

The properties of a graph can provide information about a given situation.



Example #1

Teacher Note: This graph should appear as sinusoidal.

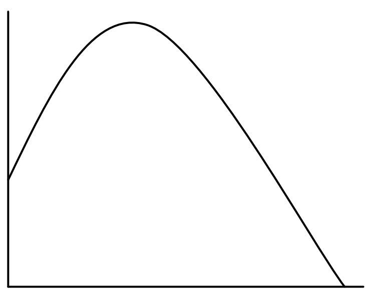


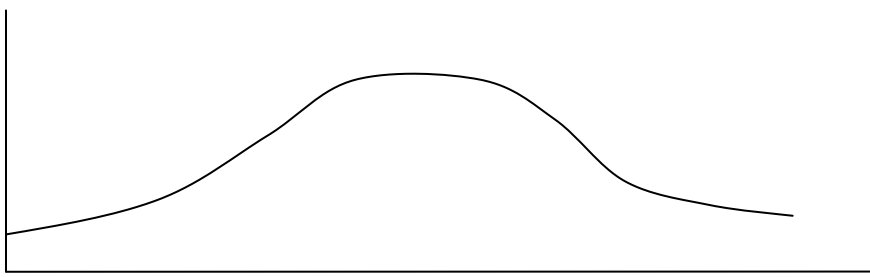
When we see graphs like this, we can describe a situation that the graph could possibly represent (there are numerous scenarios that could be described, as long as they can be justified). For example, the above graph could represent

a) the height a person off of the ground while they were riding on a ferris wheel.
b) The time of the sunrise throughout the years
c) The height of the tide during a week

Example #2

Describe a situation that the following graph could represent, and label it with appropriate dependent and independent variables:



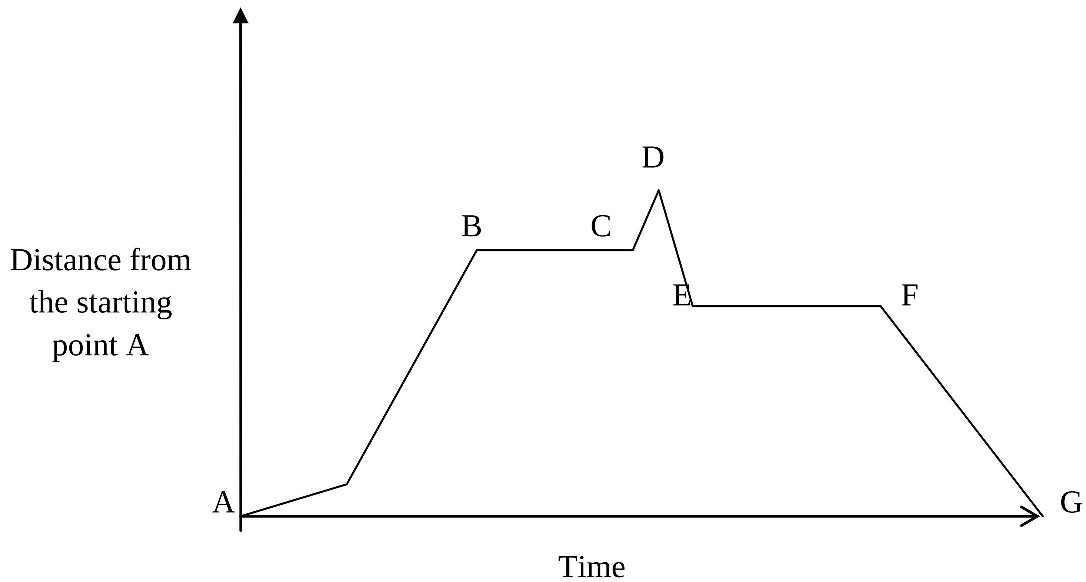


A couple include:
Health vs. Age
Height of a thrown ball vs. Time

Mean Temperature in Sher. Pk. Vs. Time of Year

Example #3

The graph shows the distance a wake boarder is from her starting point on a lake. Describe what the wake boarder could possibly be doing between and at the various points.



AB: Since the distance is increasing, the wakeboard rider is moving away from her starting point. The change in distance starts slowly at first, it then reaches a constant rate

BC: Since the distance is not changing, the rider has either stopped or is on a path that keeps her at a constant distance from the starting point.

CD: The change in direction increases so that the wakeboard rider is moving away from her starting point at a quicker rate.

DE: Since the distance is decreasing quickly, the rider is moving toward the starting point at a fast rate.

EF: Since the distance is not changing, the rider has either stopped or is on a path that keeps her at a constant distance from the starting point

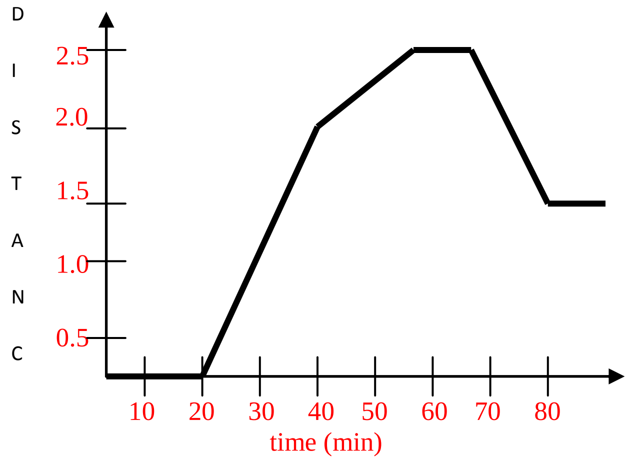
FG: The distance is decreasing to zero. The rider is returning to the starting point at a constant rate.

Example #4

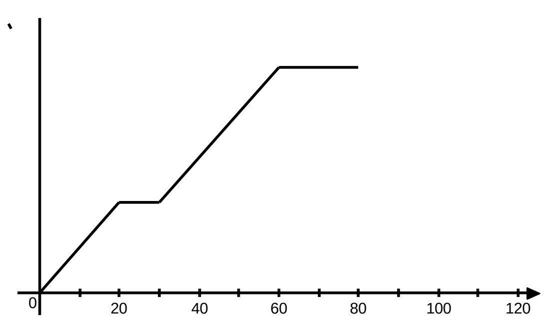
Draw a graph that coordinates with a person going for a jog. Use the following information to help create your graph:

This is the path of a person out for a jog:

- 1st 20 minutes: getting ready for the jog and then stretching
- 2nd 20: jogs away from the house for 2.0 km
- next 10–15 minutes: person starts to walk.
- Next 10 minutes: when 2.5 km from home, they stop to rest.
- Next 15 minutes: starts to jog home
- Next 10 minutes: When 1.5km from home, the jogger stops again.

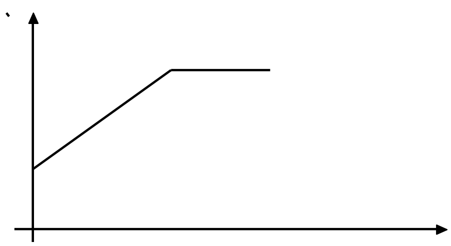


Jeff leaves home and rides his bike to visit his grandparents. From his house, he bikes directly north for 20 minutes. He then stops for 10 minutes to have lunch. He keeps heading north for another 30 minutes when he hits a bowling alley and decides to play a few games. This takes him 20 minutes. He then travels directly home which takes him 30 minutes. Assuming he travels at a constant rate, sketch a graph of his distance away from home vs time.

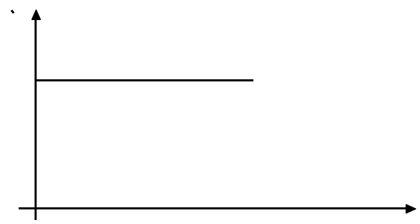


2. The following graphs represent the distance you are from school. Give an explanation for each.

a.



b.



Two explanations

– going to a friend's house after school then going home.

– walking in a circle around the school and then going into the school.

Assignment: pg. 281–283 #1–11, 13–17

Lesson 3 – Domain & Range

Domain → the set of the first elements in a certain number of specific ordered pairs of a relation

**** Domain ALWAYS refers to the *x* coordinates of ordered pairs****

Range → the set of the second elements in a certain number of specific ordered pairs of a relation.

**** Range ALWAYS refers to the y coordinates of ordered pairs****

eg.) For the relation {(2,4), (4,8), (6,12), (8, 16)}:

D: { 2, 4, 6, 8 }
R: { 4, 8, 12, 16 }

Is this a relation or a function? Function – all values of x (inputs) are different.

Example #1

For each relation below:

- Determine whether the relation is a function.
- Identify the domain and range of each relations that is a function

a.

{(7, 2), (17, 2), (18, 3), (21, 2), (17, 3)}

This is not a function because the input value of 17 has two output values of 2 and 3.

D: {7, 17, 18, 21}
R: {2, 3}

Graphing

A *scatterplot* is the result plotting data on a Cartesian (coordinate) plane that can be represented as ordered pairs.

When graphing, if the domain and range are only "elements" of the graph, we DO NOT connect the points on the graph. If the domain or range is all the real numbers (R), then we connect the points and draw arrows at both ends of the line.

eg.) {(2,5), (4,7), (5,9), (9,12)} —— We would NOT connect the points

$y = 3x + 1$ —— We would connect the points (since x could represent any Real number)

Example #2

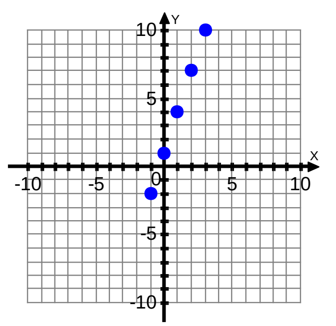
Graph the equation $y = 3x + 1$ for the domain {–1, 0, 1, 2, 3}, and for the domain of the Real numbers (R).

** Need to set up a Table of Values**

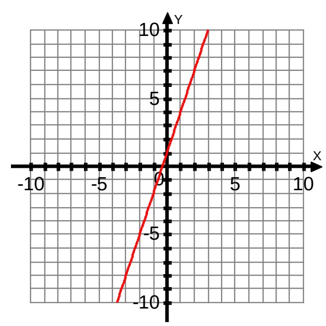
x	**3*x*+1**	**y**	**(*x*, *y*)**
–1	3(–1) + 1	–2	(–1, –2)
0	3(0) + 1	1	(0, 1)
1	3(1) + 1	4	(1, 4)
2	3(2) + 1	7	(2, 7)
3	3(3) + 1	10	(3, 10)

D: {–1, 0, 1, 2, 3}

D: Real Numbers



Created with a trial version of Advanced Grapher - http://



Created with a trial version of Advanced Grapher - http://

Have students try: $y = x + 4$ D:{–3,–2,–1,0,1}

Discrete vs. Continuous

Discrete Graph→ a graph that is a series of separate points

→ **the points are NOT connected on the graph**

Eg) The number of books in the library

The number of students in a class

Options on rolling a die

Continuous Graph→ a line graph in which the line in unbroken

→ **the points on the graph ARE connected**

Eg) Time is continuous

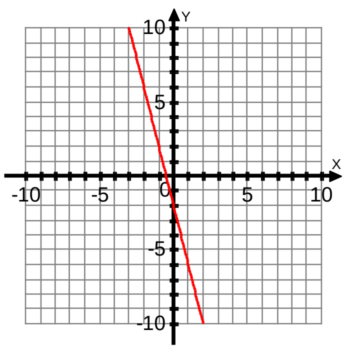
In the above example, the first graph is **discrete** (D: {–1, 0, 1, 2, 3}) , and the second graph is **continuous** (D: Real Numbers).

eg.) "If I leave here at Noon and drive to Calgary, I arrive at 3:00pm. Where was I at 1:30pm? Did I get straight to Red Deer? No." I drove everywhere in between, therefore this is **continuous** (time is continuous) because at any specific time I was at a specific point on the journey.

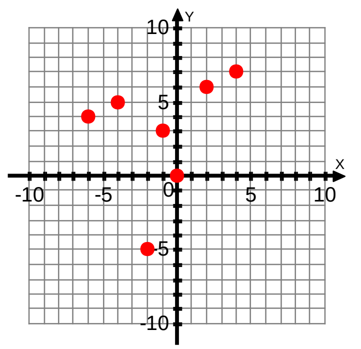
"If I count the number of students I talk to during the day…I talk to one…then two…etc. Can I talk to 4.23 students? No." There is no in between…therefore, this is **discrete.**

<u>Example #3</u>

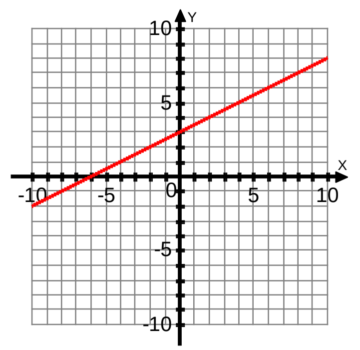
Are the following graphs discrete or continuous?



Created with a trial version of Advanced Grapher - http://



Created with a trial version of Advanced Grapher - http://



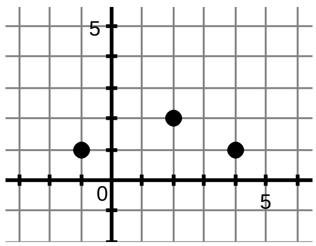
Created with a trial version of Advanced Grapher - http://

Continuous Discrete Continuous

Determine the domain and range for the following:

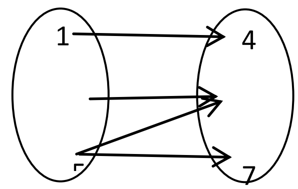
a. $\{(5, 0), (5, 1), (5, 2), (5, 3)\ldots\}$ D: $x = 5$ R: $y \in W$

b.



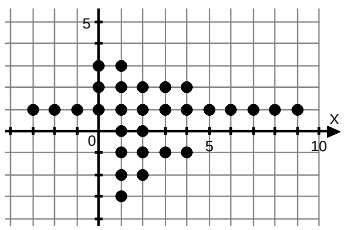
D: $\{-1, 2, 4\}$ R: $\{1, 2\}$

c.



D: $\{-5, 1, 8\}$ R: $\{-9, 4, 7\}$

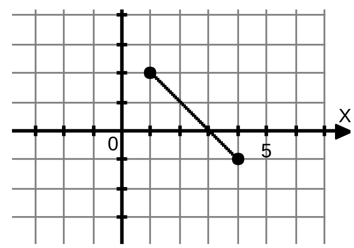
d.



D: $\{-3 \le x \le 9, x \in I\}$

R: $\{-3 \le y \le 3, y \in I\}$

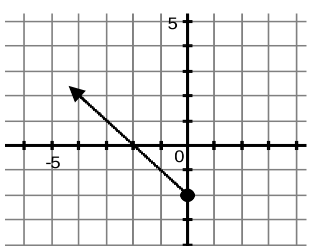
e.



D: $\{1 \le x \le 4\}$

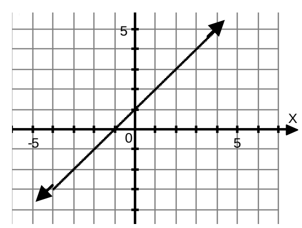
R: $\{- \le y \le 2\}$

f.



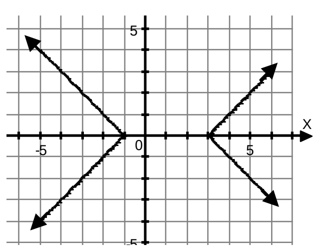
D: $\{x \le 0\}$ R: $\{y \ge -2\}$

g.



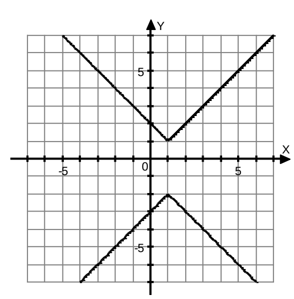
D: $\{x \in R\}$ R: $\{y \in R\}$

h.



D: $\{x \le -1 \text{ or } x \ge 3\}$ R: $\{y \in R\}$

i.

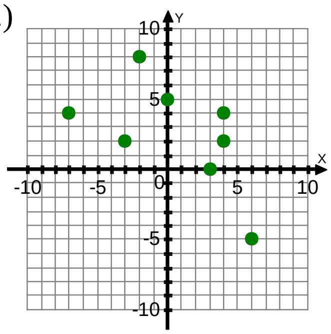


D: $x \in R$ R: $y \ge 1$ or $y \le -2$

Example #4

Determine the domain and range of the following graphs and whether they are functions or relations:

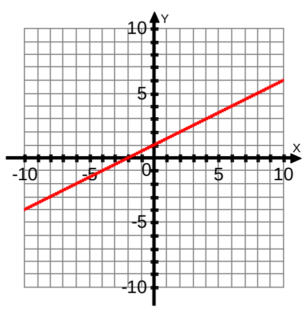
a.)



Created with a trial version of Advanced Grapher - http://

D: $\{-7, -2, -3, 0, 4, 6, 3, 4\}$
R: $\{4, 8, 2, 5, 2, -5, 0, 4\}$
Relation

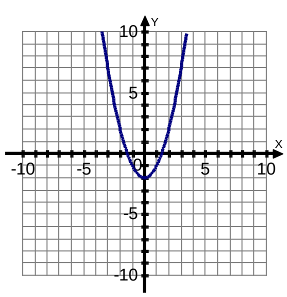
b.)



Created with a trial version of Advanced Grapher - http://

D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$
Function

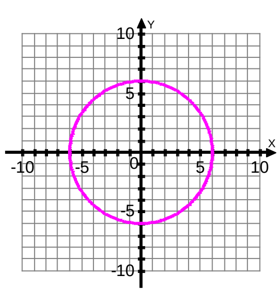
c.)



Created with a trial version of Advanced Grapher - http://

D: $x \in \mathbb{R}$
R: $y \geq -2$
Function

d.)



Created with a trial version of Advanced Grapher - http://

D: $-6 \leq x \leq 6$
R: $-6 \leq y \leq 6$
Relation

e.) Show an example where the domain could be $-2 \leq x \leq 6$, and the range $0 \leq y \leq 4$.

Assignment: Pg. 271 #5, 8, 9, 10 (only do domain and range of each)
Pg. 293–297 #1–11, 14, 16, 17

Ch5 Domain & Range

Lesson 4 – Function Notation

– replaces y.

- – read as “f at x” or “f of x”.
- – is used for functions only. (functions have only one y for each x)

If an equation can be written in the form y = (with no ±), then the relation is a function.

f(3) = 5 means that y = 5 when x = 3.

finding f(–2) means finding the value of y when x = –2.

Example #1

If $f(x) = 3x + 4$, find *f*(3), *f*(0), and *f*(–10).

f(3) = 3(3)+4
= 9+4
=13

f(0) = 3(0)+4
= 0+4
= 4

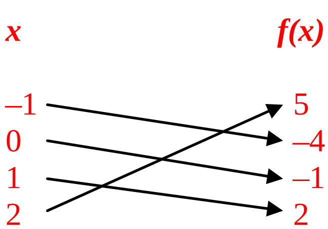
f(–10) = 3(–10)+4
= –30+4
= –26

Example #2

The function $f(x) = 3x - 1$ has a domain of {–1,0,1,2}. Draw an arrow diagram that represents the function and find the range.

First create a Table of Values:

x	3*x*–1	$f(x)$
–1	3(–1) – 1	–4
0	3(0) – 1	–1
1	3(1) – 1	2
2	3(2) – 1	5



R:{–4,–1,2,5}

Example #3

If f(x) = 5x – 6. Find:

a. f(4)   Insert 4 into f(x)

$$f(4) = 5(4) - 6$$
$$f(4) = 14$$

b. f(–2)   Insert –2 into f(x)

$$f(-2) = 5(-2) - 6$$
$$f(-2) = -16$$

c. f(0)

$$f(0) = 5(0) - 6$$
$$f(0) = -6$$

d. f(–3.2)

$$f(-3.2) = 5(-3.2) - 6$$
$$f(-3.2) = -22$$

e. f(x) = 4

$$f(x) = 4$$
$$4 = 5x - 6$$
$$10 = 5x$$
$$x = 2$$

If $g(x) = 3x^2 - 2$, find:

$g(2)$

$=3(2)^2-2$
$=(3)(4)-2$
$=12-2$
$=10$

$g(-4)$

$= 3(-4)^2-2$
$= (3)(16)-2$
$= 48-2$
$= 46$

The value of x if $g(x) = 25$

$25 = 3x^2 - 2$
$25 + 2 = 3x^2$
$27 = 3x^2$
$(27/3) = x^2$
$9 = x^2$
$x = \pm 3$

If $f(x) = 2x^2 - 3x + 1$. Find:

a. f(0)

$$f(0) = 2(0)^2 - 3(0) + 1$$
$$f(0) = 0 - 0 + 1$$
$$f(0) = 1$$

b. f(2)

$$f(2) = 2(2)^2 - 3(2) + 1$$
$$f(2) = 8 - 6 + 1$$
$$f(2) = 3$$

c. $f(-1)$

$$f(-1) = 2(-1)^2 - 3(-1) + 1$$
$$f(-1) = 2 + 3 + 1$$
$$f(-1) = 6$$

d. $f\left(\sqrt{2}\right)$

$$f\left(\sqrt{2}\right) = 2\left(\sqrt{2}\right)^2 - 3\left(\sqrt{2}\right) + 1$$
$$f\left(\sqrt{2}\right) = 4 - 3\sqrt{2} + 1$$
$$f\left(\sqrt{2}\right) = 5 - 3\sqrt{2}$$

Example #4

Write the following in function notation:

a.) $C = 0.1d + 1.2$

$C(d) = 0.1d + 1.2$

b.) $R = 1200s + 300$

$R(s) = 1200s + 300$

Example #5

Write the following as equations in 2 variables:

a.) $V(t) = 19t - 7$

$V = 19t - 7$

b.) $A(b) = 0.5b + 21$

$A = 0.5b + 21$

Example #6

Two variables are connected to the equation $t^2 + w = 25$.

a.) Express w as a function of t so that $w = w(t)$
b.) Find $w(3)$
c.) Determine the value of t for which $w(t) = 9$

a.) Solve the equation for w:

$t^2 + w = 25$
$w = 25 - t^2$ → this equation gives a rule for finding w as a function of t, so we replace w with $w(t)$.

So: $w(t) = 25 - t^2$

b.) $W(3) = 25 - (3)^2$
$= 25 - 9$
$= 16$

c.) $w(t) = 25 - t^2$
$9 = 25 - t^2$
$t^2 = 25 - 9$
$t^2 = 16$
$t = \pm 4$

Example #7

The distance a blue whale is from shore (in metres) is given by the function
$f(t) = 5t + 37$, where t is the time in seconds. Find:

a. the position of the whale at the beginning. ($t = 0$) **ANS:** 37 m

b. the position of the whale after 15 seconds. **ANS:** 112 m

c. the time when the whale is 142 m from shore. **ANS:** 21 seconds

Assignment: Pg. 271–272 # 6–7, 14–19
Optional (good for review and honours) assignment follows:

I. **$f(x) = 3x + 7$**; $g(x) = -6x + 3$; **$h(x) = -3x^2 + 5$**; $m(x) = 3^{x-1} - 2$; $p(x) = x^2 - 5x$; $t(x) = \frac{x-7}{x+3}$;

Find:

1. $f(2x - 7)$	**ANS:** $\{6x - 14\}$	2. $f(x) + 2g(x)$	**ANS**: $-9x + 13\}$
3. $h\left(2\sqrt{2}\right)$	**ANS:** -19	**4. $h(x + 3)$**	**ANS**: **$-3x^2 - 18x - 22$**
5. $p(2x - 5)$	**ANS:** $4x^2 - 30x + 50$	6. $m(1)$	**ANS**: -1
7. $m(-1)$	**ANS:** $\frac{-17}{9}$	8. $-3f(-2)$	**ANS**: -3
9. $g(2x) + f(3x)$	**ANS:** $-3x + 10$	10. $t(4)$	**ANS**: $\frac{-3}{7}$
11. $t(-3)$	**ANS:** $\varnothing$	12. $p\left(-\sqrt{7}\right)$	**ANS:** $7 + 5\sqrt{7}$

II.

1. $f(x) = 2x^2 + 3$. Find:

   a. $f(2a)$ **ANS:**$8x^2 + 3$ b. $f(n - 1)$ **ANS:**$2n^2 - 4n + 5$

   c. $f(3m + 1)$ **ANS:**$18m^2 + 12m + 5$

2. $f(x) = -3x^2 - 2x + 1$. Find $f(a + 1)$. **ANS:** $-3a^2 - 8a - 4$

3. $f(x) = \frac{3x^2 - x}{x+2}$. Find $f(-5)$. **ANS:** $\frac{-80}{3}$

4. **$g(x) = 4^{x-1} + 7$**. Find:

   a. $g(4)$ **ANS: 71** b. $g(1)$ **ANS:** 8

5. $f(x) = \frac{41x + 11}{x}$. Find $f(-3)$. **ANS:** $\frac{112}{3}$

Lesson 5 – Graphing Linear Relations/Equations

In order to graph linear equations, all we need to do is find 2 points on the graph and connect them with a straight edge. There are various methods we can use to determine 2 points on a graph. We will use the various methods in the following examples.

Example #1

Use any two points on the graph.

To determine the two points, input any value of the domain into the equation, and solve it for the output value.

Graph $y = 3x - 4$

Since $x \in \mathbb{R}$, we can choose any values of x that we would like. Try and choose two values that are smaller. This way they will be easier to work with, as well as, they will potentially be closer to the origin of the coordinate (Cartesian) plane.

Let's choose $x = 2$, and $x = 4$

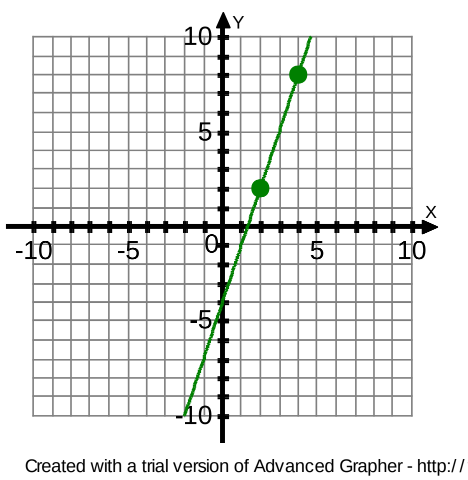
(2, ?) and (4, ?)

Substitute (input) these x values into the equation and solve for y (output).

When $x = 2$, then $y = 2$. Therefore, one point on the graph is (2, 2)

When $x = 4$, then $y = 8$. Therefore, another point on the graph is (4, 8)

Plot these 2 points on the graph and connect them with a straight edge. **Since $x \in \mathbb{R}$, then we must either draw arrows on the ends of ours graphs, or make sure the line (graph) is drawn to the end of grid that you are graphing on.**



Example #2

Use the *x* and *y* intercepts.

In order to find the *x* and *y* intercepts of a linear equation, simply let the opposite variable equal zero (0) in the equation, and solve for the remaining variable.

NOTE: *x*–intercept = "horizontal intercept"
y–intercept = "vertical intercept"

x–intercept – let $y = 0$

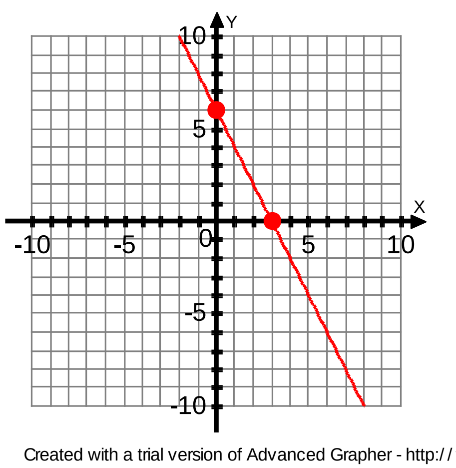
y–intercept – let $x = 0$

Graph $y = -2x + 6$

x–intercept – $0 = -2x + 6$
$-6 = -2x$
$x = 3$ Therefore: (3, 0) is one point.

y–intercept – $y = -2(0) + 6$
$y = 0 + 6$
$y = 6$ Therefore: (0, 6) is another point.

Plot these two points on the Cartesian plane and connect them!



Example #3

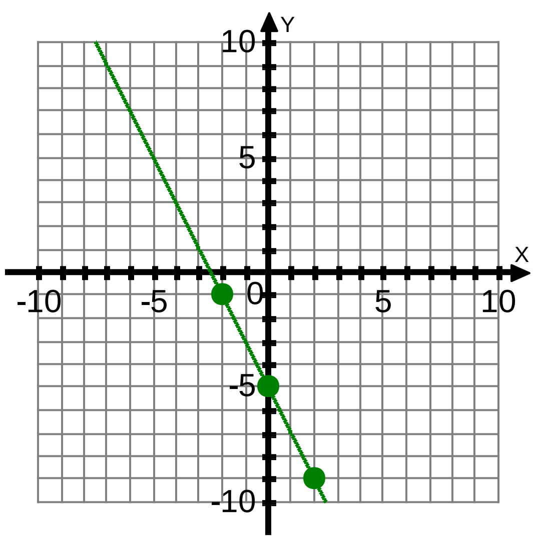
Use a Table of Values.

Construct a table of values. Since $x \in \mathbb{R}$, we can choose any values of x that we would like. Try and choose two values that are smaller. This way they will be easier to work with, as well as, they will potentially be closer to the origin of the coordinate (Cartesian) plane.

Graph $y = -2x - 5$

x	$\boldsymbol{y = -2x - 5}$	***y***	***(x, y)***
–2	$y = -2(-2) - 5$	–1	(–2, –1)
0	$y = -2(0) - 5$	–5	(0, –5)
2	$y = -2(2) - 5$	–9	(2, –9)

Now plot the points (essentially we only need two) on the Cartesian plane and connect the points.



Created with a trial version of Advanced Grapher - http://

CLASS ASSIGNMENT **NC = try without a calculator** **C = try with a calculator**

	Technique	Equation	NC or C
1.	2 points	y = 2x – 6	(NC)
2.	x & y–intercepts	3x + y = 9	(C)
3.		x = 5	(NC)
4.		y + 2 = 0	(NC)
5.	x & y–intercepts	7x + 3y + 9 = 0	(NC)
6.		–5x + y = 15	(C)
7.	x & y–intercept	2x – 3y + 7 = 0	(C)

Extra:

1.	x and y–intercepts	3x + 13y = 12	(NC)
2.		5x – 2y – 8 = 0	(C)
3.		x = 4	
4.		y + 3 = 0	

Assignment: Linear Graphing Assign.doc

Lesson 6– Clean up day.

Review Exercises

Binary Relations.jpg #1–10

Linear Relations.jpg #1–6

General Relations.jpg #1–6

Functions.jpg #1–6, 8–12, 15–17, 19–20, 22–23

Applications of Linear Functions.jpg #1–9

Lesson 7 – Applications of Relations & Rate of Change

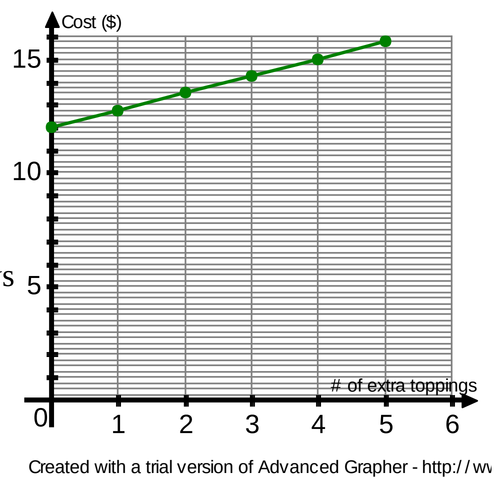
Textbook Examples:

The table of values shows the cost of a pizza with up to 5 extra toppings.

Number of Extra Toppings	**Cost ($)**
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

Have the students graph the data being sure to label the axes: (independent variable on the x–axis and dependent variable on the y–axis)

The line connecting this data is called a "trend "line as it does not really exist but shows the pattern we want.



What patterns do you see in the table?

Have the students write a rule for the pattern that relates the cost of a pizza to the number of its toppings.

$$C = 0.75t + 12.00,\ t \in W$$

How can you tell from the table that the graph represents a linear function?

Handout 1–cm grid paper and have them do the following activity:

Use the pattern of rectangles on the top of page 301. This pattern continues.

a.) Draw the next two rectangles in the pattern. Copy and complete each table of values for the next 6 rectangles.

Width of Rectangle (cm)	**Area (cm^2)**
1	
2	
3	
4	
5	
6	

Width of Rectangle (cm)	**Perimeter (cm)**
1	
2	
3	
4	
5	
6	

b. Which table of values represents a linear relation? How can you tell?

The perimeter is a linear relation, constant change in both x and y.

The area is not linear (quadratic)

c. Graph the data in each table of values. Does each graph represent a linear relation? How do we know?

The cost for a car rental is $60, plus $20 for every 100km driven. The independent variable is the distance driven and the dependent variable is the cost. We can identify that this is a linear relation in different ways.

Table of Values:

Distance (km)	Cost ($)
0	60
100	80
200	100
300	120
400	140

What changes do we see in the distance and cost?

Are they constant?

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

We can see these patterns using table of values, ordered pairs or graphs.

What characteristics can you see in the graph of a linear relation?

Graph is a line.

We can use each representation to calculate the **rate of change**. The rate of change can be expressed as a fraction:

$$\frac{\boldsymbol{change\ in\ dependent\ variable}}{\boldsymbol{change\ in\ independent\ variable}} = \frac{\$\mathbf{20}}{\mathbf{100}\boldsymbol{km}} = \$\mathbf{0.20}/\boldsymbol{km}$$

The rate of change is $0.20/km. For each additional 1km driven, the rental cost increases by 20 cents. The rate of change is constant for a linear relation.

We can determine the rate of change from the equation that represents the linear function.

Write an equation for this linear relation:

C = 0.20d + 60

Notice that the rate of change is the coefficient on the variable for distance.

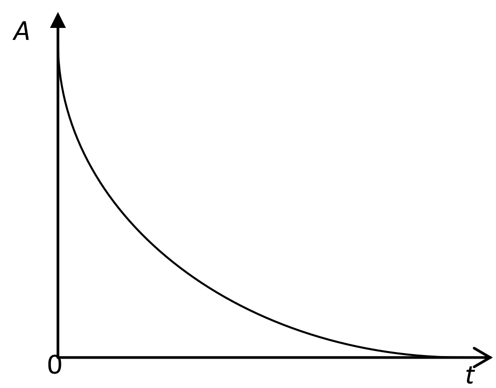
Ex. Consider each relation. Determine whether the relation is linear. Explain why or why not.

a.

x	**y**
–9	–10
–7	–5
–5	0
–3	5
–1	10

It is linear because there is a constant change in both x and y.

b. The graph shows the relationship between the amount, *A*, of a radioactive isotope present and the age of a rock sample over time, *t*, years.



Not linear because it is not a line.

c. Graph each equation:

i) $x = -2$

ii) $y = x + 25$

iii) $y = 25$

iv) $y = x^2 + 25$

Which equations represent linear relations? How do you know?

i) $x = -2$ ii) $y = x + 25$ iii) $y = 25$

The graphs are all lines.

Ex. Which relation is linear? Justify your answer.

a. A dogsled moves at an average speed of 10km/h along a frozen river. The distance travelled is related to time.

Linear, create a table of values.

1hr: 10km

2hrs: 20km

3hrs: 30km…

b. The area of a square is related to the side length of the square.

Not linear, it is quadratic.

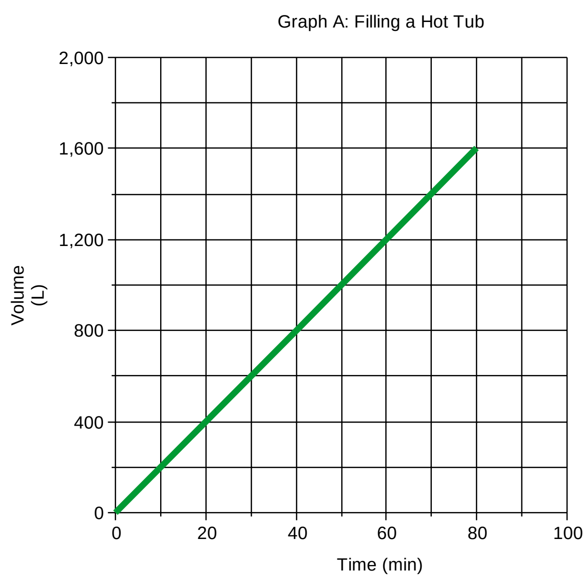
Length of 2 = area of 4

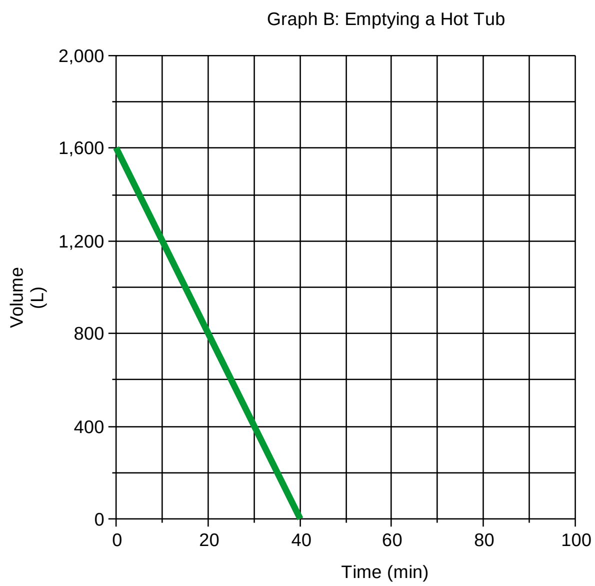
Length of 3 = area of 9

Length of 4 = area of 16

Both parts are not constant in their change.

Ex. A hot tub contains 1600L of water. Graph A represents the hot tub being filled at a constant rate. Graph B represents the hot tub being emptied at a constant rate.





a. Identify the dependent and independent variables in each graph.

Graph A: dependent: Volume, independent: time

Graph B: dependent: Volume, independent: time

b. Determine the rate of change of each relation, then describe what it represents.

Graph A: 20L/min (800/40)—represents that the tub is being filled at a constant rate of 20L every minute.

Graph B: –40L/min (800/20)——represents that the tub is draining at a constant rate of 40L every minute.

c. State the domain and range for both graphs.

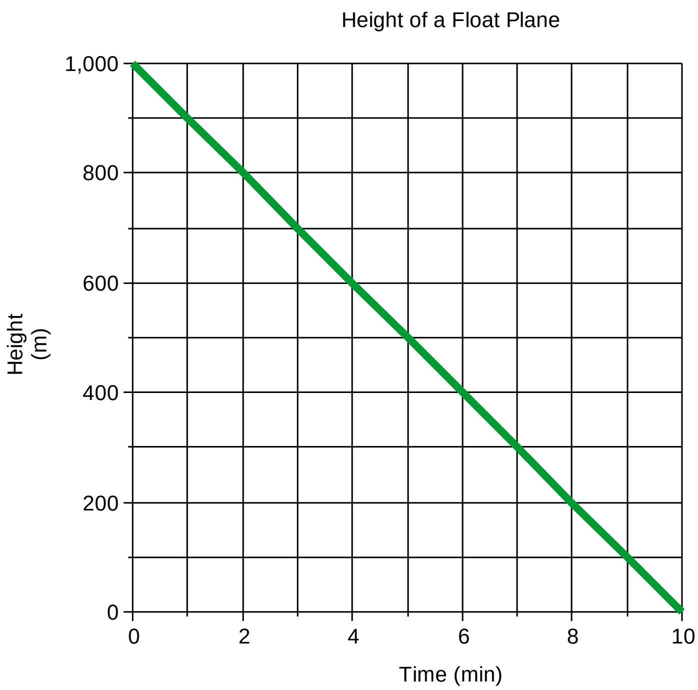
Graph 1: $D:\{0 \le t \le 80\}$ $R:\{0 \le V \le 1600\}$

Graph 2: $D:\{0 \le t \le 40\}$ $R:\{0 \le V \le 1600\}$

Assignment: pg. 307–310 #1–10, 12–19

Lesson 8 – Interpreting Graphs of Linear Functions

Textbook example: Float planes fly into remote lakes in Canada's Northern wilderness areas for ecotourism. This graph shows the height of a float plane above a lake as the plane descends to land.



Where does the graph intersect the vertical axis? What does this point represent?

1000m – Height of the plane as it begins decent.

Where does the graph intersect the horizontal axis? What does this point represent?

10 min. – It takes the plane 10 minutes to land.

What is the rate of change for this graph? What does it represent?

–100 metres/min. – Every minute the plane descends 100 metres.

What would the domain and range of this graph?

$D: 0 \leq t \leq 10$

$R: 0 \leq H \leq 1000$

The x–coordinate of the point where a graph intersects the x–axis is called the ***x–intercept*** or the **horizontal intercept.**

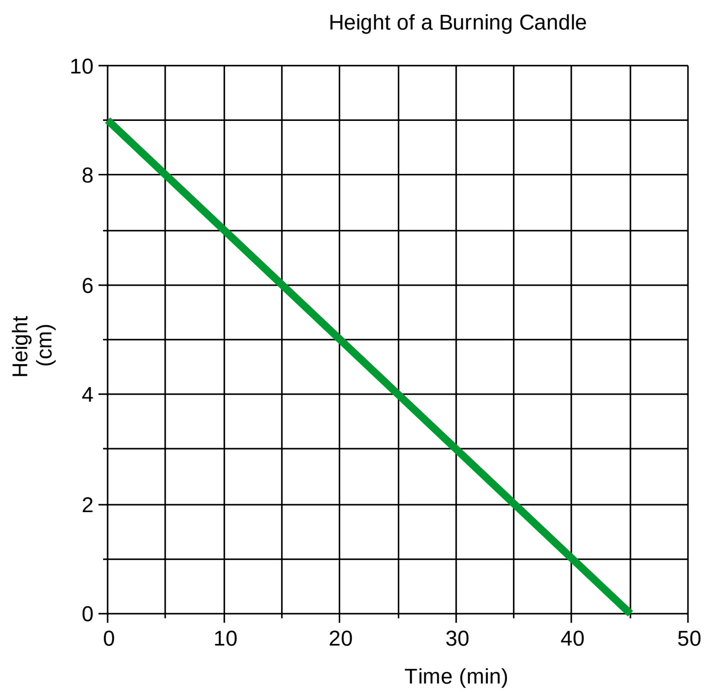
The y–coordinate of the point where a graph intersects the y–axis is called the ***y–intercept*** or the **vertical intercept.**

Have students create a graph that gives an example of a positive rate of change (labeling the independent and dependent variables):

Have them create a graph that demonstrates a negative rate of change (labeling the independent and dependent variables):

Any graph of a line that is not vertical represents a function. We call these functions linear functions.

Ex. This graph shows how the height of a burning candle changes with time.



a. Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

(0, 10), 10: y–intercept

(45, 0): 45: x–intercept

b. What are the domain and range of this function?

D: $0 \leq t \leq 45$

R: $0 \leq h \leq 10$

We can use the intercepts to graph a linear function written in function notation.

To determine the *y*–intercept, evaluate *f(x)* when *x=0*; that is evaluate *f(0).*

To determine the *x*–intercept, determine the value of *x* when *f(x) = 0.*

Ex. Sketch a graph of the linear function $f(x) = 4x - 3$

Determine the x–intercept: y = 0

$0 = 4x - 3$

$3 = 4x$

$$\frac{3}{4} = x - intercept$$

Coordinate: $\left(\frac{3}{4}, 0\right)$

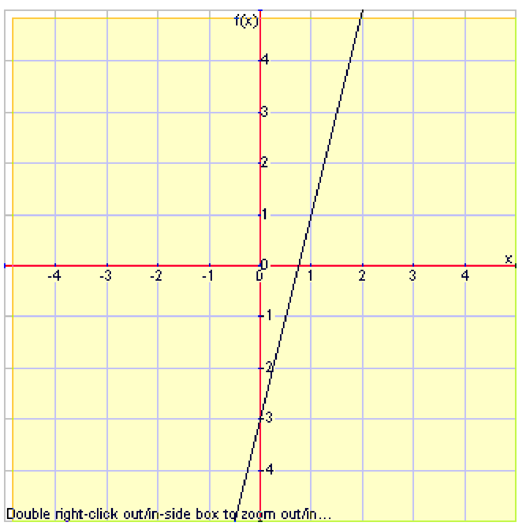
Determine y–intercept: x = 0

$y = 4(0) - 3$

y–intercept is –3

coordinate: (0, –3)

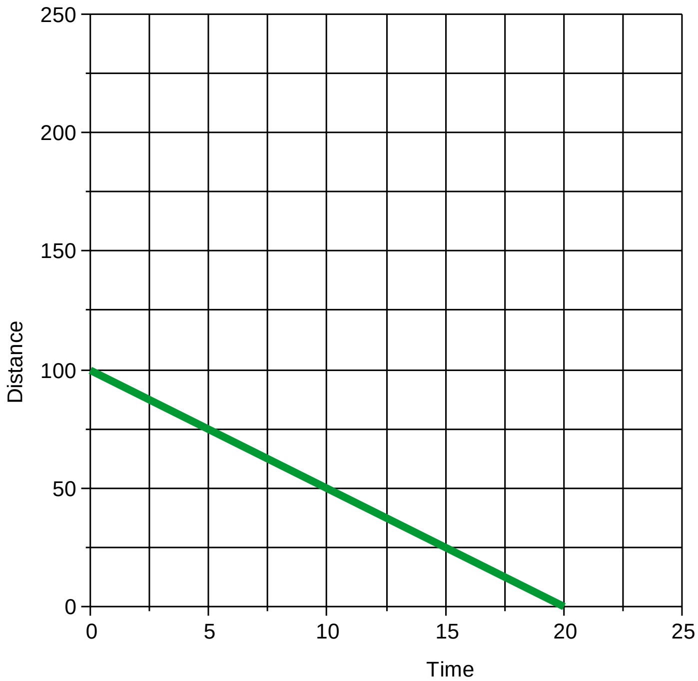
Plot the two intercepts:



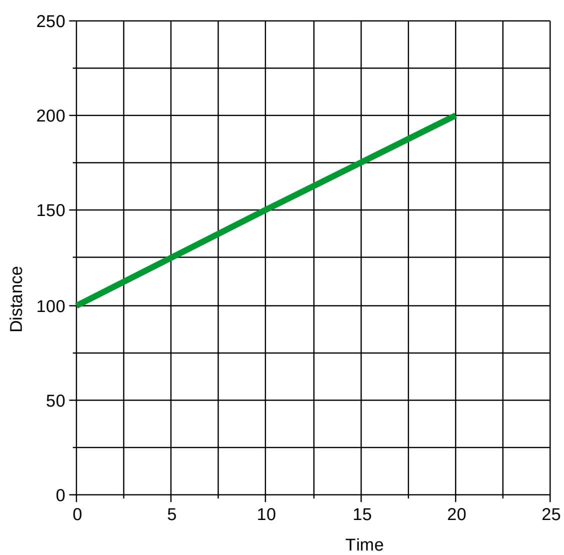
Introduce y = mx + b

Ex. Which graph has a rate of change of –5 and a vertical intercept of 100?

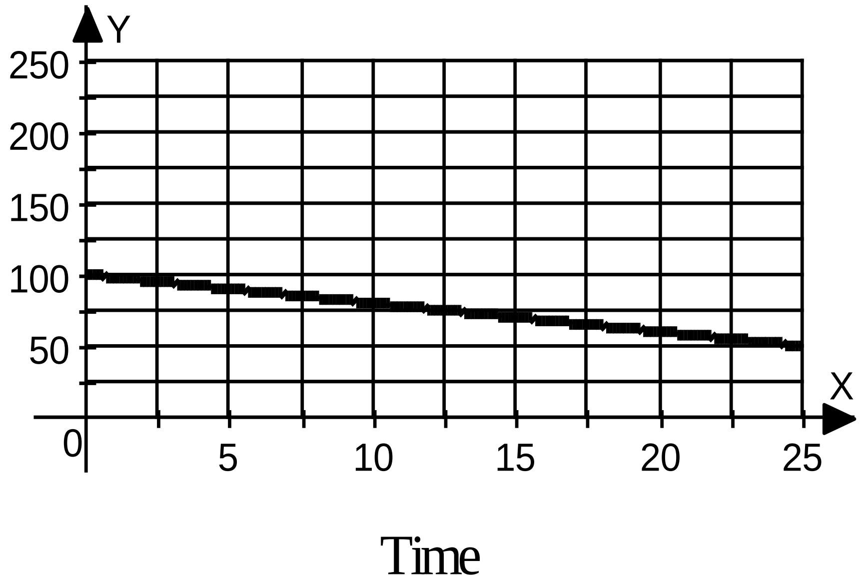
a)



b)

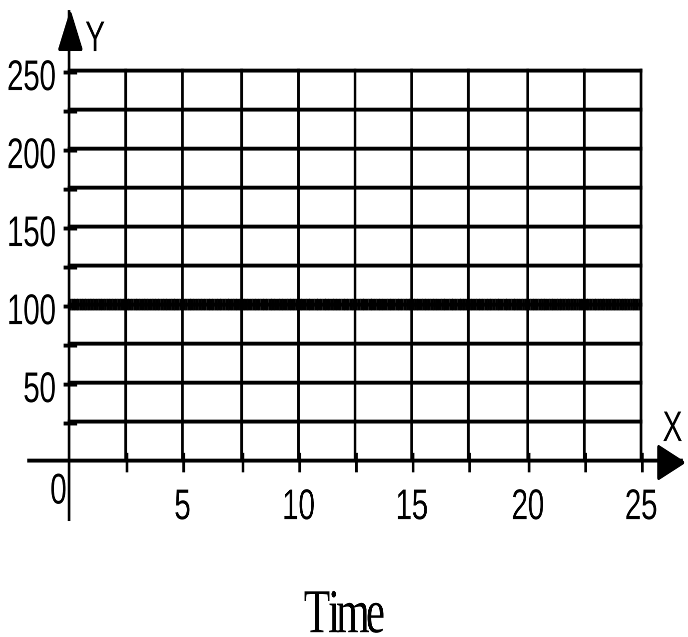


c)



Created with a trial version of Advanced Grapher - htt

d)

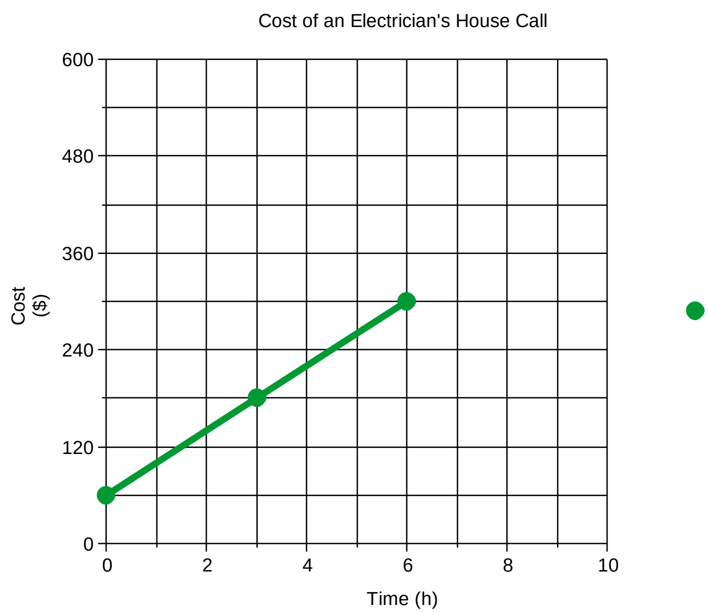


Created with a trial version of Advanced Grapher - htt

Graph in part a

Why???

Ex. This graph shows the total cost for a house call by an electrician for up to 6h work.



The electrician charges $190 to complete a job. For how many hours did she work?

$60 automatic charge (vertical intercept)

At 3hours it cost $180.

$180 – $60 = $120

$120/3 = $40/h with initial $60 cost

$190 = 40x + 60$

$190 - 60 = 40x$

$130/40 = x$

3.25h

Assignment: pg. 319–323 #1–12, 14–16, 18, 19