CHAPTER 7– SYSTEMS

Day 1

Systems of Equations
- two or more equations with 2 or more variables
- usually the number of equations equals the number of variables

Solution to a System of Equations
- value(s) of variables must satisfy all equations

Recall linear equations:

$2x+2=14$
$2x=12$
$x=6$ (one solution)

Whereas $x+y=6$ has 2 variables having many solutions {(1,5), (2,4), (3,3)} are just a few.

Example:

1. Which of the following is a solution to the system y = 3x + 5 and 6x – 5y = –43?

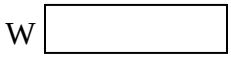
   a. (1, 8) b. (–5, 3) c. (–2, 11) d. (2, 11) Answer D (2, 11)

   For a solution to be valid, it must satisfy both equations simultaneously.
   i.e. $(1,8)$ works in the first equation but not the second.

Solving Systems
- graphically
- algebraically (elimination and substitution)

Use a diagram to model an example

1. The perimeter of a sheet of paper is 38 inches. The length is 3 inches more than its width. Bob measured the paper to be 11x 8 . Use the linear system from #1 to see if Bob is correct.



2L + 2W = 38 ∴ **2(11)+2(8) = 38**
L = 3 +W

Go over example 1,2,3 pg 397–400 text

Assignment p 401 # 4, 5, 6,8, 9,12,13,14 Honours 15–18

Day 2 Solve Systems Graphically (No calculators)

Solving Systems (Linear) Graphically

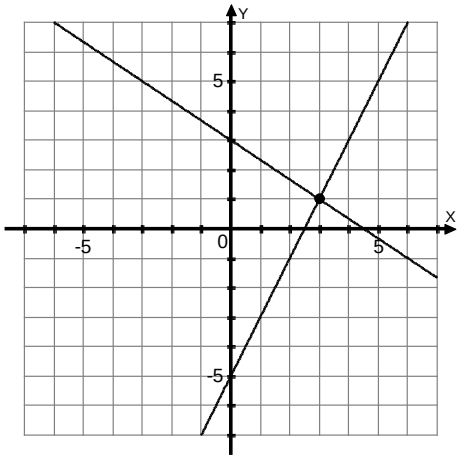
SHOW GRAPHING SYSTEMS USING SLOPE YINTERCEPT/ X+ Y INTERCEPT BEFORE USING TECHNOLOGY !!!!

1. graph each line.
2. find point of intersection from graph.

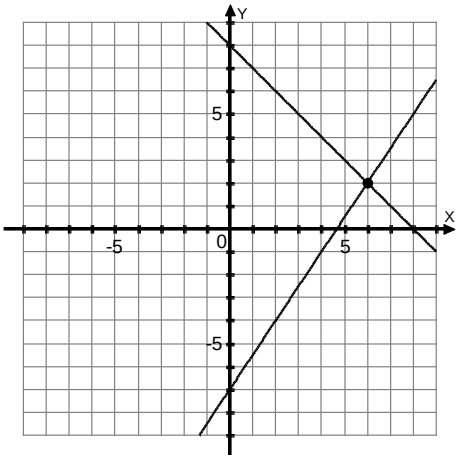
Do : Examples 1,2,3 Text p 405,406

Hand out rulers and graph paper. Use slope and y-intercept or X + Y intercepts to graph.

1. $y = 2x - 5$
   $2x + 3y = 9$ **ANS:** $x = 3, y = 1 \rightarrow (3, 1)$

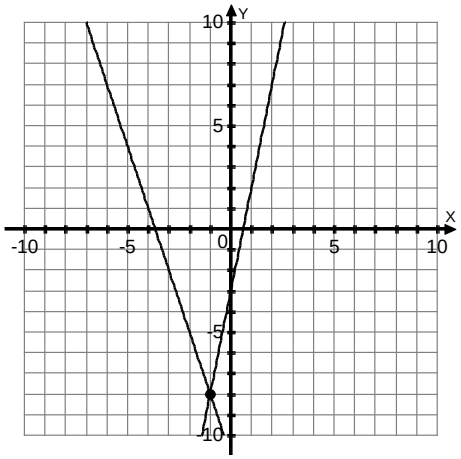


2. $y = 8 - x$
   $14 = 3x - 2y$ **ANS:** $x = (6,2)$



3. $y = 5x - 3$
   $3x + y + 11 = 0$

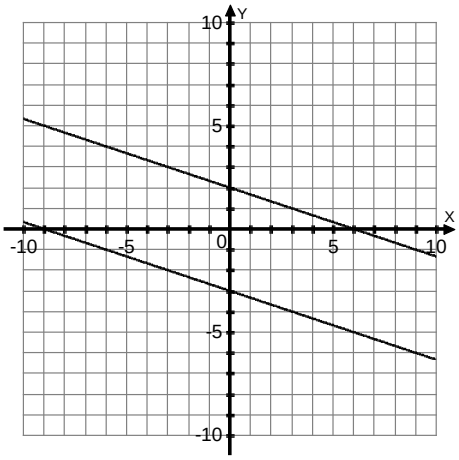
ANS: $(-1, -8)$



4. $y = -\frac{1}{3}x + 2$
   $6y + 2x = -1$

ANS: no solutions { } or ϕ

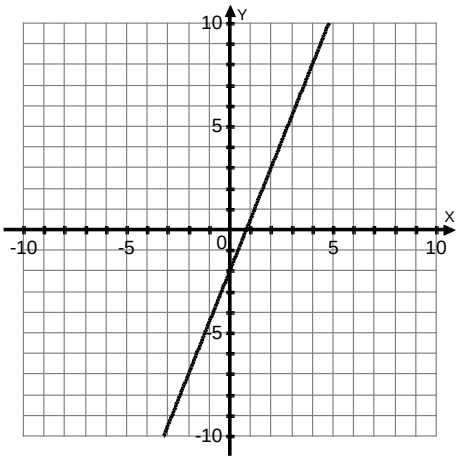
Notice how these two lines are parallel. Thus, they never cross. As a result, there is no solution to this system. We write empty brackets to mean, empty set or no solution.



5. $10x - 4y = 8$
   $y = \frac{5}{2}x - 2$

ANS: $\left\{(x, y) \middle| y = \frac{5}{2}x - 2\right\}$

This means the two equations are equal i.e. one line is on top of the other. Thus, they cross an infinite number of times.



More Examples:

1. $y = 3x - 6$
   $y = \frac{1}{3}x + 2$ **ANS:** (3,3)

2. $y = -2x + 5$
   $y = -2x - 3$ **ANS:** Lines are parallel, no solution.

3. $y = -2x + 7$
   $y = -2x + 7$ **ANS:** $\{(x, y) | y = -2x + 7\}$

Assignment

Pg. 409–410 #3 b,d 4b , 5 all 6, 7 b,d, 8, 9, 11,12, 15,16 Honours 17,18

DAY 3

Solving Systems Graphically (use calculator)

Steps
1. type each equation into y = on you calculator
   equation 1 into $y_1 =$
   equation 2 into $y_2 =$
2. ensure your window is such that you can see the two lines cross.
3. hit 2nd Calc (Trace)
4. hit 5 intersect
5. hit enter 3 times. You now have the point of intersection. (1st curve, 2nd curve matters only if you have more than 2 equations).

Examples:

NOTE: If the point of intersection is above or below the window, it is not necessary to adjust the window. If the point of intersection is right or left of the window it must be adjusted?

1. x+ y = 2 — Rearrange to $y = -x + 2$
   2x – y = 7 — Rearrange to $y = 2x + 7$
   ANS: (3, –1)

2. y = 2x –5 — Equation is in y = format
   x + y = 8 — Rearrange to $y = -x + 8$
   ANS: (4.3, 3.7)

3. 2x + y = 3 — Rearrange to y = –2x +3
   3x + 2y = 11 **ANS:** (–5, 13) — Rearrange to $y = \frac{-3}{2}x + \frac{11}{2}$ (the window does not need to be adjusted)

4. 3x +5y = 5 — Rearrange to $y = \frac{-3}{5}x + 1$ *or* $y = (-3x + 5)/5$
   2x + y = 22 — Rearrange to y = –2x + 22
   Adjust your window to see the point of intersection!
   ANS: (15, –8)

Assignment
p. 415 7.1,7.2,7.3 plus worksheet

HONOURS CLASSES!!
EXTRA

1. $y = 2^x - 5$
   $y = x^2 + 2$
   ANS: (5, 27)

2. $y = 0.6x^2 - 7x + 3$
   $y = x - 5$
   ANS: (1.09, –3.91), (12.24, 7.24)

3. $y = |0.5x - 5| - 4$
   $y = -\frac{1}{3}x + 7$
   ANS: (19.2, 0.6), (–36, 19)

4. $y = \sqrt{7 - x}$
   $x + 4y + 10 = 0$
   ANS: (–36.33, 6.58)

5. $y = x^3 - 5x$
   $2y - x + 6 = 0$
   ANS: (0.58, –2.71), (–2.58, –4.29), (2, –2)

DAY 4 + half of day 5

Substitution Method

The goal is to get one of the equations to either $y =$ or $x =$.

You then substitute that equation into the other equation and solve for the remaining variable.

View a **YOUTUBE video** on the substitution method http://www.youtube.com/watch?v=dJsL4_LkJYc

Examples:

1. y = 5x – 1
   3x + y = 15

   ANS: (2, 9)

   The first equation is in y = form. Substitute equation 1 into equation 2.

   $3x+(5x-1)=15$
   $8x=16$
   $x=2$

   Once you have x, solve for y.

   $y=5x-1$
   $y=5(2)-1$
   $y=9$

   The solution to the system is (2,9)

2. x = 3y – 7
   3x + 2y = –10

   ANS: (–4, 1)

   The first equation is in x = form. Substitute equation 1 into equation 2.

   $3(3y-7)+2y=-10$
   $9y-21+2y=-10$
   $11y=11$
   $y=1$

   Once you have y, solve for x.

   $x=3y-7$
   $x=3(1)-7$
   $x=-4$

3. 3x – y = 7
   12x = 4y + 3

   ANS: $\{\ \}$

   Get equation 1 into y = form $3x-7=y$

   Substitute the new equation 1 into equation 2.

   $12x=4(3x-7)+3$
   $12x=12x-28+3$
   $0=-25$

When you work down to $0 =$ constant (other than 0), you have parallel lines. No solution.

4. $p = \frac{2}{3}m + 5$

$4m - 6p + 30 = 0$

ANS: $\left\{(m, p) \middle| p = \frac{2}{3}m + 5\right\}$ This means infinite solutions. Eqn 1 = Eqn 2

Equation 1 is in p = form so substitute into equation 2.

$4m - 6\left(\frac{2}{3}m + 5\right) + 30 = 0$

$4m - 4m - 30 + 30 = 0$

$0 = 0$

When you work down to $0 = 0$, you have the same line. ∞ solutions,

5. $\frac{x}{4} + \frac{y}{2} = -3$

$x + \frac{y}{5} = 6$

ANS: (8, −10)

Multiple equation 1 by 4 and equation 2 by 5 to remove all fractions.

$4\left[\frac{x}{4} + \frac{y}{2} = -3\right]$ $x + 2y = -12$

$5\left[x + \frac{y}{5} = 6\right]$ $5x + y = 30$

Now get equation 1 to x = or equation 2 to y = Both options work.

$x = -2y - 12$

$5(-2y - 12) + y = 30$

$-10y - 60 + y = 30$

$-9y = 90$

$y = -10$

$x = -2y - 12$

$x = -2(-10) - 12$

$x = 20 - 12$

$x = 8$

6. $6y = 6x - 19$
$0 = 15 - 5x - y$

ANS: $\left(\frac{109}{36}, \frac{-5}{36}\right)$

Get equation 2 to y =
$y = 15 - 5x$

$6(15 - 5x) = 6x - 19$
$90 - 30x = 6x - 19$
$109 = 36x$
$x = \frac{109}{36}$ You can type this fraction into your calc. to get y.

$y = 15 - 5x$
$y = 15 - 5\left(\frac{109}{36}\right) = \frac{-5}{36}$

7. $\frac{1}{2}x - \frac{4}{5}y = -2$ **ANS:** $\left(\frac{-23}{3}, \frac{-55}{24}\right)$
$y = \frac{1}{4}x - \frac{3}{8}$

Review
When you work down to $0 =$ constant (other than 0), you have parallel lines. No solution. Question 3.
When you work down to $0 = 0$, you have the same line. ∞ solutions. Question 4.

PROBLEM APPLICATION EXAMPLES

Example 1

Betty and Dick are off to the movies with some friends, sadly they do not drive so they have to take the bus.

Betty buys 5 admission tickets and 3 bus tickets . She pays $65.00. Dick buys 2 admission tickets and 1 bus ticket. He only pays $25.00. Set up a system and determine the price of each ticket.

Solution

5a + 3b = 65
2a + b = 25
The above system can be solved using elimination (See Day 5) or substitution. Given that the second equation has only one b, let's use substitution by isolating b.

b = –2a +25 Substitute into equation 1

$$5a+3(-2a+25)=65$$
$$5a-6a+75=65$$
$$-a=-10$$
$$a=10$$

If a = 10, insert 10 into either equation to get b.

Solution (a=10,b=5)

Example 2

Montey purchased $ 20,000 in two types of savings bonds. One bond earned him 8 % annually and the other bond earned 5% annually. The total interest earned was $ 1360. How much money was invested at each rate?

x + y = 20,000
0.08x +0.05y = 1360

Solution x= $12,000
y=$8,000

Multiply equation 2 by 100 to get rid of the decimals.
8x + 5 y = 136000

Isolate x or y in equation 1. I shall isolate x.
x = –y + 20000

Use the substitution method.

$$8(-y+20000)+5y=136000$$
$$-8y+160000+5y=136000$$
$$-3y=-24000$$
$$y=8000$$

If y = 8000, then x = 12000

Assignment p 425 4 b,c, 5 a,d 6 a,b 8, problems 11, 12 15, 16 18, 19 a,d

plus: HONOURS

1. $y = 5x - 3$
   $3x + y + 11 = 0$

   ANS: $(-1, -8)$

2. $x = 2y - 7$
   $5x - 10y + 17 = 0$

   ANS: $\{\ \}$

3. $3a - b = 10$
   $2b = 6a - 20$

   ANS: $\{(a, b) | b = 3a - 10\}$

4. $\frac{x}{5} - \frac{y}{2} = 5$
   $x + y = 4$

   ANS: $(10, -6)$

5. $2x = y + 6$
   $2x + 5y = 6$

   ANS: $(3, 0)$

DAY 5 Elimination Method

View a **YOUTUBE video** on the elimination method http://www.youtube.com/watch?v=AZeMPzxqPRg

The goal is to get the additive inverse for one variable in the system.
Decimals are acceptable (not ideal) but avoid fractions!
Get all equations to the form $Ax + By = C$ **Standard Form**

Examples:

1. $2x - y = 10$
   $5x + y = 11$

   ANS: $(3, -4)$

   The negative y in equation 1 is opposite the positive y in equation 2.
   Add the two equations together.

   $2x - y = 10$
   $5x + y = 11$
   $7x = 21$
   $x = 3$

   Once you have x, insert into either original equation to get y.

   $2(3) - y = 10$
   $6 - y = 10$
   $-y = 4$
   $y = -4$

2. $4x - 3y + 11 = 0$
   $5x + 6y = -4$

   ANS: (–2, –1)

   Get equation 1 to Ax + By = C form.
   $4x - 3y = -11$
   $5x + 6y = -4$

   Multiply equation 1 by 2.

   $8x - 6y = -22$
   $5x + 6y = -4$
   $13x = -26$
   $x = -2$

3. $2c + 5d = 17$
$7c = 3d + 2$

ANS: $\left(\frac{61}{41}, \frac{115}{41}\right)$

Get equation 2 to Ac + Bd = C form $7c - 3d = 2$

$2c + 5d = 17$
$\underline{7c - 3d = 2}$

Multiply equation 1 by 3 and equation 2 by 5.

$6c + 15d = 51$
$\underline{35c - 15d = 10}$
$41c = 61$
$c = \frac{61}{41}$

Use calculator and insert c into either original equation to get d.
or use elimination again to solve for d (eliminate c)

4. $2x - 5y - 11 = 0$
$-4x + 10y - 3 = 0$

ANS: $\{\ \}$

$2x - 5y = 11$
$-4x + 10y = 3$

$4x - 10y = 22$
$\underline{-4x + 10y = 3}$
$0 = 25$

This means no solution.

5. $\frac{x - 3}{2} = \frac{y}{5} + 3$
$3(x - 4) - (7 - 4y) = 8$

ANS: (9, 0)

Multiply equation 1 by 10. Lowest common multiple of 2 and 5.
Clean up equation 2 by removing brackets.

Equation 1	$5x - 15 = 2y + 30$	*becomes*	$5x - 2y = 45$
Equation 2	$3x - 12 - 7 + 4y = 8$	*becomes*	$3x + 4y = 27$

Multiply equation 1 by 2 to get –4y
Equation 2 is unchanged

$10x - 4y = 90$
$\underline{3x + 4y = 27}$
$13x = 117$
$x = 9$
$y = 0$

6. $0.3x - 0.5y = 1.2$
$0.7x - 0.2y = -0.1$

ANS:$(-1, -3)$

Multiply each equation by 10 to get rid of the decimals. You can work with the equations as they are but integers are easier to work with.

$3x - 5y = 12$
$7x - 2y = -1$

You can eliminate either the x or the y.

To get rid of x, multiply equation 1 by –7 and equation 2 by 3. Get –21x and 21x.
To get rid of y, multiply equation 1 by –2 and equation 2 by 5. Get 10y and –10y.

Additional Examples:

1. $5(m - 3) + 2(n + 4) = 10$
$3(m + 4) - 4(n + 4) = -21$ **ANS:** (1, 6)

2. $4x - 5 = 2y$
$1 = 5y - 10x$ **ANS:** $\{\ \}$

3. $\frac{x+2}{6} - \frac{3(y+2)}{2} = 1$
$\frac{x-2}{2} + \frac{y-1}{3} = 0$ **ANS:** (4, –2)

Assignment

Pg 437 # 3 a,c #4 b, #6 b,d #7 b , #8, 10, 12 a, d # 13, 17, 19 Honours # 22,24
plus:

1. $2x = 7y - 13$ and $11x - 5 = 3y$ **ANS:** $\left(\frac{74}{71}, \frac{153}{71}\right)$

DAY 6
Review and more problems

Problem Solving

Examples:

1. The perimeter of a rectangle is 70 m. 5 times the width is 7 more than twice the length. Find the dimensions.

$$2W + 2L = 70$$
$$5W = 2L + 7$$

ANS: W = 11 m, L = 24 m

2. The population of Smoky Lake is 5 times the population of Bellis. Their total population is 1266. Find the population of each.

$$S = 5B$$
$$S + B = 1266$$

ANS: S = 1055, B = 211

3. John invests $30 000; some at 4% and the remainder at 7%. If he earned $1 560, how much did he invest at each rate?

$$x + y = 30\,000$$
$$0.04x + 0.07y = 1\,560$$

ANS: x = $18 000, y = $12 000

4. Tom has $330 in $5 and $20 bills. He has 24 bills in total. How many of each?

$$x + y = 24$$
$$5x + 20y = 330$$

ANS: x = 10, y = 14

5. You have a 80% vinegar solution and a 30% solution. You want 20L of a 65% solution. How many litres of each solution would you need?

	80%	30%	65%
Litres	x	y	20
Vinegar	0.8x	0.3y	$0.65 \times 20 = 13$

ANS: x = 14, y = 6

6. Bill took 8 hours to travel 93 km. Part of the trip he biked at 15 km/h and the remainder he walked at 6 km/h. How far did he bike? walk?

	bike	walk
d	15x	6y
r	15	6
t	x	y

$$15x + 6y = 93$$
$$x + y = 8$$

ANS: biked 75 km, walked 18 km

Assignment – Problems worksheet doc.

DAY 7

Properties of Systems of Linear Systems

We know that two lines have three potential scenarios:
1. They cross once and have one solution.
2. They are parallel, thus never cross and thus have no solution.
3. They are the same line, thus they cross an infinite number of times.

Each of these scenarios has a name. Let's look at their names.

1. **One Solution – Independent System—intersect , only one solution**

   $y = 2x–5$
   $2x + 3y = 9$ **ANS: x = 3, y = 1 (3, 1)**
   Independent System, one solution

2. **No Solution– { } – Inconsistent System (Parallel lines)**

   y = $-\frac{1}{3}x + 2$ **ANS:** No Solutions { }
   $6y + 2x = – 1$ parallel lines

3. **Infinite Number of Solutions ∞ Same line – Dependent (also known as Coincident)**

   $10x – 4y = 8$ **ANS:** $\left[\left\{(x, y)\middle| y = \frac{5}{2}\middle| x – 2\right\}\right]$

   y = $\frac{5}{2}x – 2$ **Same Line , infinite number of solutions**

Try :

4. $y = x– 3$ (nearest hundredth) **ANS:** x = $\frac{–5}{7}(0.71)$ $y = \frac{–26}{7}(3.71)$
   $y = 8x +2$ Independent

5. $2x + y = –10$ **ANS:** (20, –50)
   $4x + y – 30 = 0$ Independent

Look at slopes and y – intercept to determine if systems are independent, inconsistent or dependent.

If slopes ar different — one solution **Independent**
If slopes are the same and y – intercept the same, same line, infinite no of solutions **Dependent**
If slopes are equal but y –int not the same, parallel lines. No solutions **Inconsistent**

Examples:

1. $y = 3x - 6$

   $y = \frac{1}{3}x + 2$

   Independent

2. $y = -2x + 7$

   $y = -2x + 7$

   Dependent

3. $y = \frac{5}{7}x + 5$

   $y = \frac{5}{7}x - 3$

   Inconsistent

Assignment

Page 448 #4 , 5 , 7, 10–12, 14,15, 22

Day 8 Problems and Review

Assign . Text p 452 – 454 # 2, 5, 7, 9 a, d, 10 b, d, 15 a, b, 20 c, d, p 455 1, 2, 4, 6 a, b plus worksheet

1. Which statement below is **false** for this linear system?

$3x - 4y = -9.5$ ①

$-2x + \frac{y}{2} = 2$ ②

A. If you multiply equation ② by 8, then add the new equation to equation ①, you can eliminate y.
B. The system has one solution because the slopes of the lines are different.
C. If you replace equation ② with $4x - y = -4$, the new system will have the same solution as the original system.
D. The solution of the linear system is: (2, –0.5)

2. Which system has **exactly one** solution? Thus, which system is **independent**?

A. $y = -4x - 2$
$y = -4x + 5$

B. $6x - 3y = -1$
$-2x + y = 4$

C. $\frac{1}{3}x + \frac{1}{2}y = 2$
$\frac{1}{6}x + y = \frac{5}{2}$

D. $y = 3x - 2$
$y = 3x + 2$

3. Solve each linear system.

a) $-3x - 6y = 9$
$2x + 2y = -4$

b) $3x - 4y = 13$
$5x + 3y = 12$

c) $\frac{1}{2}x - \frac{1}{3}y = \frac{5}{12}$
$\frac{5}{6}x + \frac{1}{2}y = \frac{1}{6}$

4. Given the linear equation $4x - 2y = -4$, write another linear equation that will form a linear system with each number of solutions. Explain what you did.

a) exactly one solution **b)** no solution **c)** infinite solutions

5. **a)** Write a linear system to model this situation:
In Claire's school, 41 of the 80 grade 10 students were not born in Canada. Sixty percent of the boys and 40% of the girls in grade 10 were not born in Canada.
b) Solve this related problem: How many boys and how many girls are in grade 10? Explain what you did.

6. A gift shop sold hand-made moccasins. One order of 4 pairs of children's moccasins and 3 pairs of women's moccasins cost $244.65. Another order of 2 pairs of children's moccasins and 4 pairs of women's moccasins cost $229.70.
a) Write a linear system to model this situation.
b) Solve this related problem: What is the cost for a pair of each type of moccasin?

Answers on next page

ANSWERS

1. D **2.** C

3. **a)** $x = -1, y = -1$ **b)** $x = 3, y = -1$

c) $x = \frac{1}{2}, y = -\frac{1}{2}$ Explanation is below:

I multiplied equation ① by 12 and equation ② by 6.
That left me with $6x - 4y = 5$ ③ and $5x + 3y = 1$ ④.
I multiplied equation ③ by 3 and equation ④ by 4 to get $18x - 12y = 15$ and $20x + 12y = 4$. I then added these equations to get $38x = 19$, or $x = \frac{1}{2}$.
I substituted this value for x in equation ④ and solved for y.
I got $y = -\frac{1}{2}$. I then verified my solution.

4. Equations may vary.

a) I wrote $4x - 2y = -4$ in slope-intercept form to identify its slope: $y = 2x + 2$
The slope of the line is 2. For a linear system with one solution, the lines must have different slopes. So, I let the second line have slope –3. Its equation is: $y = -3x + 2$

b) For a linear system with no solution, the lines must be parallel. The lines must have equal slopes but different y-intercepts. The equation $y = 2x + 2$ has slope 2 and y-intercept 2. The second line must also have slope 2. Let its y-intercept be –2.
Its equation is: $y = 2x - 2$

c) For a linear system with infinite solutions, the lines must be coincident. The equations must be equivalent.
To determine an equivalent equation, I multiplied the given equation by 2: $2(4x) - 2(2y) = 2(-4)$
Its equation is: $8x - 4y = -8$

5. **a)** Let b represent the number of boys in grade 10 and g represent the number of girls in grade 10. The linear system that models the situation is:

$b + g = 80$ ①
$0.60b + 0.40g = 41$ ②

b) There are 45 boys and 35 girls in grade 10. I eliminated b by multiplying equation ① by 0.6 and subtracting the new equation from equation ②. I got $0.2g = 7$, or $g = 35$. I then substituted this value into equation ① and solved for b. I got $b = 45$. I then verified my solution.

6. **a)** Let c dollars represent the cost of a pair of children's moccasins and let w dollars represent the cost of a pair of women's moccasins.

$4c + 3w = 244.65$
$2c + 4w = 229.70$

b) \$28.95 for children's moccasins and \$42.95 for women's moccasins

Day 9 Quiz

Day 10 Review

Day 11 Unit Exam